ANALYSIS OF PLANE POROUS EMITTERS WITH SURFACE COMBUSTION AND A HEATED ARTICLE

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Computational dependences are obtained for porous emitters, taking account of the influence of the velocity and heat of combustion of the injectant, the thermophysical properties, the porosity of the packing, and the degrees of the interacting media.

In connection with the extensive practical application of porous emitters with surface combustion, it is expedient to give a method to analyze them, and all the more so since there is very little published on this question [1-3]. All the fundamental parameters governing the operation of a porous emitter, whose diagram is represented in Fig. 1, are taken into account in the method presented in this paper.

The fuel injectant (liquid or gas) with initial temperature T_ε is filtered in the direction from the "cold" wall surface ($y = y_1$) of thickness $l = y_2 - y_1$ to the "hot" ($y = y_2$) surface with a transverse flux density j_s of the substance (see Fig. 1). Thin-walled articles fabricated from metal, e.g., and moving at a definite velocity v_3 are located at a distance $l_g = y_3 - y_2$ from the "hot" surface, are heated from the emitter and the gas layer to a given temperature T_{3f}, and are heat insulated from each other.

To analyze the heat-transfer process with radiation between the diffusing parallel infinite surfaces taken into account, the differential equations characterizing the stationary temperature distribution $t = T/T_\infty$ in the porous body and the injectant must be solved, which have the following form in dimensionless variables (here and henceforth, the prime denotes the derivative with respect to $\bar{y} = y/y_2$) [4]:

$$t'' - \xi t' + Q = 0, \tag{1}$$

$$t_s'' - \xi_s t_s' = 0 \tag{2}$$

with the following boundary conditions

$$\bar{y} = -\infty,\ t_s = t_\varepsilon; \tag{3}$$

$$\bar{y} = \bar{y}_1,\ t_s = t = t_1,\ t' = \lambda_{s\Sigma} t_s'; \tag{4}$$

$$\bar{y} = 1,\ t = t_g = t_2,\ t' - \lambda_{g\Sigma} t_g' = q_s \xi - q_3^l; \tag{5}$$

$$\bar{y} = \bar{y}_3,\ t = t_3,\ V_3 \Delta t_3 = -\lambda_{g\Sigma} t_g' + q_2^l. \tag{6}$$

Here

$$\lambda_{s\Sigma} = \frac{\lambda_s}{\lambda_\Sigma};\ \lambda_{g\Sigma} = \frac{\lambda_g}{\lambda_\Sigma};\ Q = \frac{q v y_2^2}{\lambda_\Sigma T_\infty};\ q_s = \frac{G Q_s}{T_\infty c_{ps}};\ \xi = \frac{j_s c_{ps} y_2}{\lambda_\Sigma};\ \xi_s = \frac{j_s c_{ps} y_2}{\lambda_s};\ V_3 = \frac{(c_p \rho v)_3}{\lambda_\Sigma y_2^{-1}};\ \Delta t_3 = t_{3f} - t_i,$$

where G is a coefficient characterizing the completeness of combustion of injectant; Q_s, heat of combustion; and v_3, velocity of motion of the article being heated. The derivative t_g' in conditions (5) and (6) is determined under the stationary conditions being considered by the relationship

$$t_g' = (t_{3m} - t_2)/(\bar{y}_3 - 1), \tag{7}$$

where

$$t_{3m} = \sqrt{t_{3f} t_i}. \tag{8}$$

The emissivity ε_g of a gas layer of thickness l_g is defined by the equality

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$$\varepsilon_g = 1 - \exp(-\tau_g), \tag{9}$$

where τ_g is the integrated optical thickness of the gas layer $(\tau_g = \varkappa l_g)$; $\varkappa$ is the absorption coefficient. Values of the radiation heat fluxes q_2^l and q_3^l in the boundary conditions (5) and (6) and taking account of the presence of the emitting and absorbing gas between the "hot" and "cold" surfaces equal

$$q_2^l = \frac{\bar{q}_2^l y_2}{\lambda_\Sigma T_\infty}, \quad q_3^l = \frac{\bar{q}_3^l y_2}{\lambda_\Sigma T_\infty}. \tag{10}$$

According to [5], here

$$\bar{q}_2^l = \frac{\sigma (T_2^4 - T_{3m}^4)}{0.75\tau_g + \varepsilon_2^{-1} + \varepsilon_3^{-1} - 1}, \quad \bar{q}_3^l = -\bar{q}_2^l, \tag{11}$$

where $\sigma = 5.67 \cdot 10^{-8}$ W/m^2•deg K^4, and $T_{3m} = t_{3m}T_\infty$.

We obtain the solution of the differential equations (1) and (2) under the boundary conditions (3)-(5) by two quadratures

$$t = F(\bar{y}) + [t_2 - F(1)] \frac{\exp \xi \bar{y} - \exp \xi \bar{y}_1}{\exp \xi - \exp \xi \bar{y}_1} + t_1 \frac{\exp \xi - \exp \xi \bar{y}}{\exp \xi - \exp \xi \bar{y}_1}, \quad \bar{y}_1 \leqslant \bar{y} \leqslant 1, \tag{12}$$

$$t_s = t_\varepsilon + (t_1 - t_\varepsilon) \exp [\xi_s (\bar{y} - \bar{y}_1)], \quad -\infty \leqslant \bar{y} \leqslant \bar{y}_1, \tag{13}$$

where

$$F(\bar{y}) = \frac{1}{\xi} \int_{\bar{y}_1}^{\bar{y}} Q(\bar{y})\, d\bar{y} - \frac{\exp \xi \bar{y}}{\xi} \int_{\bar{y}_1}^{\bar{y}} Q(\bar{y}) \exp(-\xi \bar{y}) d\bar{y};$$

$$F(1) = F(\bar{y})\big|_{\bar{y}=1}. \tag{14}$$

For $Q(\bar{y})$ = const, Eq. (12) becomes

$$t = \frac{Q}{\xi}(\bar{y} - \bar{y}_1) + \left[t_2 - \frac{Q}{\xi}(1 - \bar{y}_1)\right] \frac{\exp \xi \bar{y} - \exp \xi \bar{y}_1}{\exp \xi - \exp \xi \bar{y}_1} + t_1 \frac{\exp \xi - \exp \xi \bar{y}}{\exp \xi - \exp \xi \bar{y}_1}. \tag{15}$$

Using the second boundary condition in (4), which characterizes the equality of the heat fluxes for $\bar{y} = \bar{y}_1$, as well as the solutions of (12) and (13), we obtain the value of the temperature t_1 on this boundary

$$t_1 = t_\varepsilon [1 - \exp \xi (\bar{y}_1 - 1)] + \exp \xi (\bar{y}_1 - 1) [t_2 - F(1)]. \tag{16}$$

Eliminating the value of t_1 from (12)-(15), we obtain from (12) and (14), respectively

$$t = F(\bar{y}_1) + t_\varepsilon [1 - \exp \xi (\bar{y} - 1)] + \varphi(\bar{y}) [t_2 - F(1)], \tag{17}$$

$$t = Q\xi^{-1} [(\bar{y} - \bar{y}_1) + \xi^{-1} - \varphi(\bar{y})(\xi^{-1} + 1 - \bar{y}_1)] + t_\varepsilon [1 - \exp \xi (\bar{y} - 1)] + t_2 \varphi(\bar{y}), \tag{18}$$

where

$$\varphi(\bar{y}) = \frac{1 - \exp \xi (\bar{y}_1 - 1)}{\exp \xi (1 - \bar{y}_1) - \exp \xi (\bar{y}_1 - \bar{y})}.$$

Using the second condition in (5) and (6), which are the heat-balance equations on the emitting and heated surfaces, we obtain the following additional algebraic equations from which the value of t_2 and the velocity V_3 of the article motion can be found which assures its being heated to a given temperature t_{3f}

$$t_2^4 E + [\xi + \lambda_{g\Sigma}/(\bar{y}_3 - 1)] t_2 = \xi (t_\varepsilon + q_s) + \int_{\bar{y}_1}^{1} Q(\bar{y})\, d\bar{y} + \lambda_{g\Sigma} t_{3m}/(\bar{y}_3 - 1) + E t_{2f}^2 t_i^2, \tag{19}$$

$$V_3 (t_{3f} - t_i) = \lambda_{g\Sigma} (t_2 - t_{3m})/(\bar{y}_3 - 1) + E (t_2^4 - t_{3f}^2 t_i^2), \tag{20}$$

where

$$E = \frac{k}{\ln\left(\frac{1}{1-\varepsilon_g}\right)^{0.75} + \frac{1}{\varepsilon_2} + \frac{1}{\varepsilon_3} - 1}; \quad k = \frac{\sigma T_\infty^3 y_2}{\lambda_т (1-\Pi)}.$$

For $Q(\bar{y}) = Q = \text{const}$, Eq. (19) is converted into an algebraic equation of the fourth degree in t_2:

$$t_2^4 E + [\xi + \lambda_{g\Sigma}/(\bar{y}_3 - 1)]\, t_2 = \xi (t_\varepsilon + q_s) + Q(1 - \bar{y}_1) + \lambda_{g\Sigma} t_m/(\bar{y}_3 - 1) + E t_{3f}^2 t_i^2. \tag{21}$$

The solution of problem (1)-(6) is the following. The value of t_2 is determined from (20) or (21) for given parameters q_s, Q, ξ, t_ε, t_i, t_{3f}, ε_2, ε_3, ε_g, $\bar{y}_1$, $\bar{y}_2$, and $\bar{y}_3$. Then the velocity V_3 is determined from (20). The temperature distribution over the thickness of the porous wall is determined from (17) or (18). Values of q_2 and q_3 can be obtained from the dependences

$$q_2 = t'(1) = \xi (t_2 - t_\varepsilon) - Q(1 - \bar{y}_1),$$
$$q_3 = V_3 (t_{3f} - t_i). \tag{22}$$

The results of a computation represented in Table 1 and in Figs. 2 and 3 were obtained for $t_i = t_\varepsilon = 1$, $\bar{y}_2 = 1$, and the following values of the other parameters, one of which was taken to be variable in order to analyze its influence on the process under consideration:

$$\bar{y}_1 = 0.8;\ y_1 = 0.08\ \text{m};\ y_2 = 0.10\ \text{m};\ y_3 = 0.15\ \text{m};\ t_{3f} = 2.638;\ q_s = 60;\ \varepsilon_g = 0.1;\ \varepsilon_2 = 0.9;\ \varepsilon_3 = 0.7;\ Q = 0;\ \xi = 1. \tag{23}$$

(The coordinate axes for both V_3 and q_2, q_2^l and q_3 coincide in Figs. 2 and 3.)

The influence of the coordinate $\bar{y}_1$ [or the wall thickness $l = 1 - \bar{y}_1$ for the segment l divided into n parts (n = 10)] is shown in Fig. 2a for the parameters (23) but for Q = 1. The maximum increase in t with the diminution in wall thickness is observed on its "cold" side for $\bar{y} = \bar{y}_1$, which corresponds to n = 0. For $\bar{y} = 1$ or n = 10 the dimensionless temperature t_2 is practically invariant since its value is determined mainly by the heat of combustion of the injectant on the emitting surface. The order of the diminution of q_2^l, q_3, and V_3 with the growth $\bar{y}_1$ is the same (see Fig. 2a). Values of q_2 increase here, which is due to the corresponding increase in the gradient t' [(1)]. The dimensionless temperature t diminishes with the growth of the emissivities ε_2 (see Fig. 2b) and ε_3 (Fig. 2c), whereas the radiation fluxes q_2^l and velocities V_3 increase. Since the change in the parameters mentioned occurs at a constant temperature of the article being heated t_{3f}, then under these conditions the radiation equilibrium in the system builds up with the growth of ε_2 and ε_3 for a corresponding diminution in the temperature t, and therefore t_2, as well as for an increase in V_3 and q_3. The radiation flux q_2^l defined by means of the relation (11) hence increases with the growth of ε_2 and ε_3 despite the diminution of t_2 and the temperature gradient t' [(1)] or q_2, since a change in t_2 is negligible. An increase in the value of t_2 with the growth of ε_g (see Table 1) occurs because of intensification of the heat supply from the emitting gas layer to the "hot" surface. A diminution in the flux q_2^l, defined by means of (11), despite the increase in t_2 is due to the appropriate influence of the value of ε_g. The obvious deduction that the velocity of its motion V_3 diminishes as the given temperature of the article being heated t_{3f} rises, but the values of t increase, follows from the data presented in Table 1. As the power of the internal energy source Q increases the values of V_3, q_3, and q_2^l also grow, while the temperature gradient in the porous wall and therefore, the flux q_2 also diminish (see Table 1).

Since combustion of the injectant occurs on the "hot" surface of the porous wall, then as the blowing parameter ξ grows (see Fig. 3a), all the dimensionless fluxes and the velocity V_3 as well as the body temperature t increase. The dimensionless heat of combustion q_s (Fig. 3b) exerts an analogous influence. The parameters q_2^l, q_3, and V_3 vary most strongly for small values of $\bar{y}_3$, i.e., for $\bar{y}_3 \leqslant 1.5$ (Fig. 3c). For $\bar{y}_3 > 1.5$ the radiation flux q_2^l, which is exponentially dependent on τ_g, $\tau_g = \varkappa(y_3 - y_2)$, varies negligibly. In order to maintain this value of t_{3f} = const with the growth of $\bar{y}_3$, the wall temperature t must be increased, as indeed follows from Fig. 3c. Since the limits of variation of $\bar{y}_3$ and t or t_2 are significant as compared to the range of ε_g, then the fluxes q_2 increase with the growth in the thickness of the gas layer thickness $l_g = y_3 - y_2$. The values of q_2^l, V_3, and q_3, which are functions of the parameters l_g, ε_g, and t_2 according to (19)-(23), diminish here for $y_3 > 1.5$, which is due to the influence of the mentioned parameters. Values of ε_g determined by the relationship

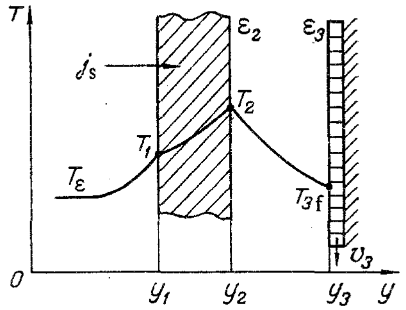

Fig. 1. Diagram, in principle, of a porous emitter.

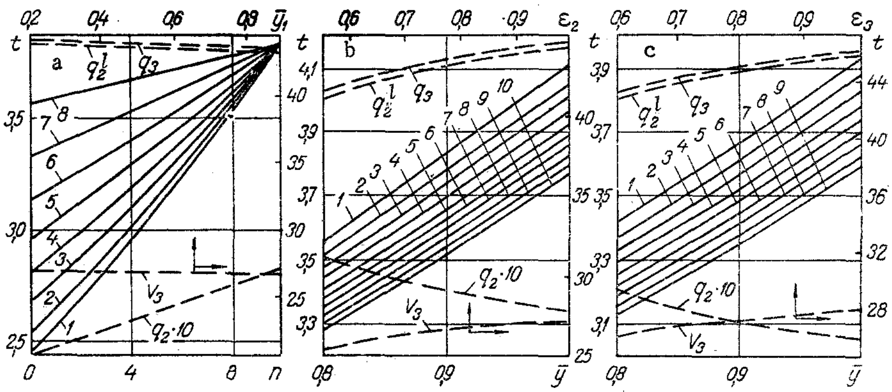

Fig. 2. Distribution of the dimensionless temperature t (solid lines) over the wall thickness $\bar{y}$ and also the dependences of the heat fluxes q_2, q_2^l, q_3 and the velocity V_3 (dashes) on the following parameters: a) on $\bar{y}_1 = 1 - \bar{l}$, where $\bar{l} = l/y_2$ is the wall thickness divided into n parts, n = 10 [1) $\bar{y}_1 = 0.2$, 2) 0.3, 3) 0.4, 4) 0.5, 5) 0.6, 6) 0.7, 7) 0.8, 8) 0.9]; b) on ε_2 [1) $\varepsilon_2 = 0.55$, 2) 0.60, 3) 0.65, 4) 0.70, 5) 0.75, 6) 0.80, 7) 0.85, 8) 0.90, 9) 0.95, 10) 1.00]; c) on ε_3 [1) $\varepsilon_3 = 0.60$, 2) 0.65, 3) 0.70, 4) 0.75, 5) 0.80, 6) 0.85, 7) 0.90, 8) 0.95, 9) 1.00].

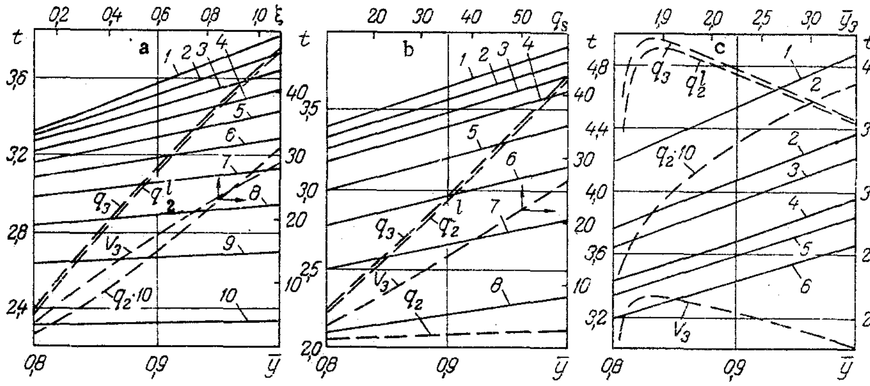

Fig. 3. Temperature t distribution over the wall thickness $\bar{y}$ (solid lines) and also the dependences of q_2, q_2^l, q_3, and V_3 on the following dimensionless quantities: a) on the blowing parameter ξ [1) ξ = 0.1, 2) 0.2, 3) 0.3, 4) 0.4, 5) 0.5, 6) 0.6, 7) 0.7, 8) 0.8, 9) 0.9, 10) 1.0]; b) on the heat of combustion of the injectant q_s [1) $q_s = 65$, 2) 60, 3) 55, 4) 50, 5) 40, 6) 30, 7) 20, 8) 10]; c) on the distance $\bar{y}_3$ [1) $\bar{y}_3 = 3.5$, 2) 2.0, 3) 1.7, 4) 1.3, 5) 1.2, 6) 1.1].

TABLE 1. Values of the Dimensionless Temperature t over the Porous Wall Thickness $\bar{y}$ and of the Velocity V_3 as Well as the Heat Fluxes q_1, q_2, q_3, and q_2^l.

Parameter being varied	$10^2 t_1$	$10^2 t$ $\bar{y}=0{,}82$	$10^2 t$ $\bar{y}=0{,}90$	$10^2 t$ $\bar{y}=0{,}94$	$10^2 t$ $\bar{y}=0{,}98$	$10^2 t_2$	$10^2 V_3$	$10^3 q_2$	$10^2 q_3$	$10^2 q_2^l$
$\varepsilon_g=0{,}005$	329	333	353	363	374	379	2715	279	4449	441
0,05	330	335	354	365	375	381	2706	281	4433	4389
0,1	332	336	356	367	377	383	2695	283	4416	4371
0,2	335	340	360	370	381	387	2673	2872	4378	4333
0,3	339	344	364	375	386	392	2648	2917	4337	4291
0,4	343	348	368	379	391	397	2620	2966	4292	4245
0,5	347	352	373	384	396	402	2589	3020	4241	4193
0,7	358	364	387	397	409	416	2508	3158	4109	4058
$t_{3f}=1{,}27$	330	335	354	364	375	381	16244	2809	4435	4382
1,61	330	335	354	365	376	381	7213	2813	4431	4381
1,96	331	335	355	365	376	382	4632	2818	4427	4378
2,30	331	336	355	366	377	382	3409	2824	4422	4375
2,64	332	336	356	367	377	383	2695	2831	4416	4371
2,98	332	337	357	367	378	384	2227	2838	4409	4367
3,32	333	338	357	368	379	385	1896	2847	4401	4361
3,66	334	338	358	369	380	386	1650	2856	4393	4354
4,34	335	340	360	371	382	388	1308	2876	4375	4339
$Q=-90$	144	147	196	243	308	347	1786	20470	2926	2889
−60	208	211	250	286	332	360	2086	14600	3417	3377
−30	270	274	303	326	355	370	2389	8720	3914	3872
0	322	336	356	367	377	383	2695	2831	4416	4371
15	362	367	382	386	388	388	−2850	117	4668	4623
30	393	398	408	406	399	393	−3004	3067	4922	3004

$$\varepsilon_g = 1 - \exp[-\varkappa y_2(\bar{y}_3 - 1)],$$

grow as $\bar{y}_3$ increases, where ε_g equals 0.18, 0.33, 0.45, 0.75, 0.86, 0.99, respectively, for $\bar{y}_3$ = 1.1, 1.2, 1.3, 1.7, 2.0, 3.5.

If it is necessary to compute the dimensionless blowing parameter ξ with the heat of combustion q_s and composition of the combustion products taken into account, then the heat-conduction coefficient of the gas layer between the emitter and the article being heated λ_g should be determined with the multicomponent mixture concentration taken into account, then the value of $\lambda_{g\Sigma} = \lambda_g/\lambda_\Sigma$ is determined for given remaining parameters in (23) from (20) or (21) in which the ξ enters.

According to [3], for a ceramic perforated emitter with surface combustion, the temperature gradient $\Delta T = T_2 - T_1$ for a 14 mm wall thickness varied between the limits 400-800°K depending on the process parameters. If we take T_1 = 300°K, T_2 = 1200°K, then for some materials used most often to fabricate porous emitters, we have the following values of the heat-conduction coefficient λ_T (W/m·deg [6]: For heat-resistant steel of the type ÉI417 we have λ_T = 14 for T_1 = 300°K and λ_T = 17 for T_2 = 1200°K, while for a Dinas brick refractory we have 0.91 and 1.14, respectively. Therefore, the difference in the values of λ_T, used in the formula $\lambda_\Sigma = \Pi\lambda_s + (1 - \Pi)\lambda_T \approx (1 - \Pi)\lambda_T$, does not exceed 25% even for the maximum gradients ΔT. In computing the temperature of a porous emitter wall by means of (17) or (18), the assumption of constancy of the thermophysical properties specifies an error not greater than 10%, which can be diminished by an appropriate selection of the governing temperature [4]. The results presented and the values of the error are valid even in a computation of λ_g.

It should be noted that all the physical hypotheses as well as the analytical dependences for the porous wall with an injectant filtered through it are completely valid even for perforated packings.

NOTATION

Π, porosity; λ, heat-conduction coefficient; c_p, specific heat at constant pressure; $\lambda_\Sigma = (1 - \Pi)\lambda_T + \Pi\lambda_s$; q_V, specific power of the internal energy sources or sinks; ρ, mass density, T, temperature. Subscripts: T, porous body skeleton; s, injectant; Σ, total (effective) quantities; i, initial; m, geometric mean; f, finite; g, gas layer; 1, "cold" wall surface; 2, "hot"; 3, heated article; ε, values as $y \to \infty$; ∞, scalar quantities.

LITERATURE CITED

1. R. Ramershwar and N. F. Volkov, Indian J. Technol., 7, No. 3, 74 (1969).
2. T. Burakovskii, E. Gizin'skii, and A. Salya, Infrared Emitters [in Russian], Énergiya, Leningrad (1978).
3. O. N. Bryukhanov, Radiation–Convective Heat Transfer during Gas Combustion in Perforated Systems [in Russian], Leningrad State Univ. (1977).
4. G. T. Sergeev, Principles of Mass and Heat Transfer in Reacting Systems [in Russian], Nauka i Tekhnika, Minsk (1976).
5. R. Siegel and J. R. Howell, Thermal Radiation Heat Transfer, McGraw-Hill (1972).
6. V. S. Chirkin, Thermophysical Properties of Nuclear Engineering Materials [in Russian], Atomizdat, Moscow (1968).

THEORY OF NONLINEAR HEAT AND MASS TRANSFER ON A POROUS SEMIINFINITE PLATE

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The nonlinear transport process (thermal conductivity or diffusion) is considered in a viscous liquid flowing near the plane of a semiinfinite plate. It is shown that under certain conditions there is rigorous spatial localization of the thermal or diffusive boundary layer.

Let the stationary flow of a Newtonian viscous liquid move over the plane of a semi-infinite plane $x \geqslant y$, $y = 0$, (Fig. 1) in the positive direction of the x axis. We assume that the velocity distribution at the external boundary of the laminar boundary layer formed over the plate is described by the expression $U = cx^m$, where c and m are constants$\geqslant 0$ (one-parameter class of boundary-layer theory [1]). For the sake of generality, it is also assumed that on the surface of the plate there is inhomogeneous fluid blowing or suction, proportional to $x^{(m-1)/2}$. It is assumed that on the surface of the plate there is heat transfer or isothermal diffusion of the plate material in the leading flow, and the corresponding transport coefficient χ depends on the transfer characteristic f(x, y) (temperature or concentration) according to the power law

$$\chi = an\left(\frac{f}{f_w}\right)^{n-1};\ a,\ n,\ f_w - \text{const} > 0.$$

Here and below the subscript w denotes the value of the corresponding quantity at the surface of the plate.

In the boundary-layer theory approximation the nonlinear transport process under consideration is described by the system of equations [2]

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \nu\frac{\partial^2 u}{\partial y^2};\quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial f}{\partial x} + v\frac{\partial f}{\partial y} = \frac{a}{f_w^{n-1}}\frac{\partial^2 f}{\partial y^2}. \tag{2}$$

Here u(x, y) and v(x, y) are the longitudinal and transverse components of the fluid velocity.

Assuming that there is no transferable characteristic in the leading flow ("vanishing background"), the boundary conditions which the solution of system (1), (2) must satisfy are written in the form

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